Data-driven controller design in the Loewner framework

Worshop MACS
Data-driven control and analysis of dynamical systems

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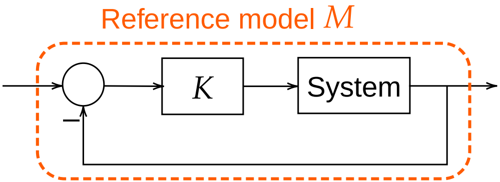
Sept 30th & Oct 1st, 2025

Reference model based data-driven control

Given data from the system P, design K such that the resulting closed-loop is as close as possible to the reference model M

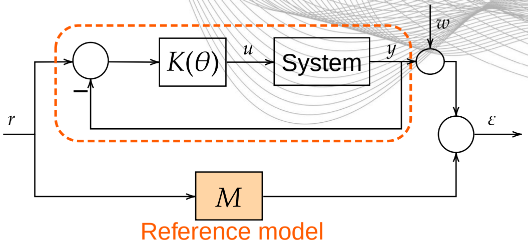


Main techniques:

- Iterative Feedback Tuning (IFT)
- Correlation-based Tuning (CbT)
- Virtual Reference Feedback Tuning (VRFT)
- Loewner Data-Driven Control (L-DDC)

Overview of these methods - CbT and IFT

$$\min_{\theta} \left\| \frac{PK(\theta)}{1+PK(\theta)} - M \right\|_2^2$$



CbT

$$\varepsilon(s) = \left(M - \frac{PK(\theta)}{1+PK(\theta)} \right) r(s) + \frac{1}{1+PK(\theta)} w(s)$$

Iterative correlation-based controller tuning, Karimi et al. (2004).

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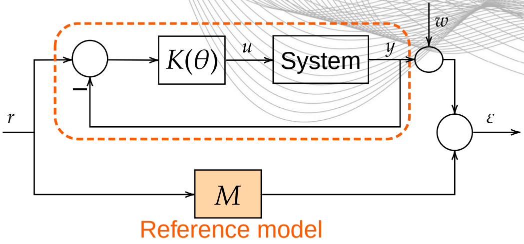

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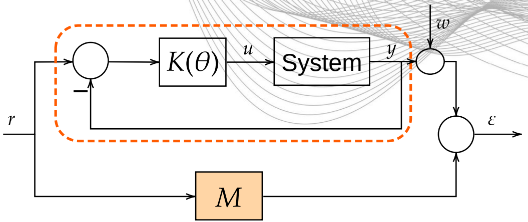

Reference model

CbT

$$\varepsilon(s) = \left(M - \frac{PK(\theta)}{1+PK(\theta)} \right) r(s) + \frac{1}{1+PK(\theta)} w(s)$$

IFT

$$\frac{\partial \varepsilon}{\partial \theta} = \frac{1}{K(\theta)} \frac{\partial K}{\partial \theta} \left(\frac{PK(\theta)}{1+PK(\theta)} (r - y(\theta)) \right)$$

Iterative correlation-based controller tuning, Karimi et al. (2004).
Iterative feedback tuning: theory and applications, Hjalmarsson et al. (1998).

Overview of these methods - VRFT and L-DDC

VRFT

$$\min_{\theta} \left\| \frac{PK(\theta)}{1+PK(\theta)} - M \right\|_2^2 \rightarrow \min_{\theta} \|u - K(\theta)r^\star\|_2^2$$

Virtual reference feedback tuning: a direct method for the design of feedback controllers, Campi et al. (2002).

Overview of these methods - VRFT and L-DDC

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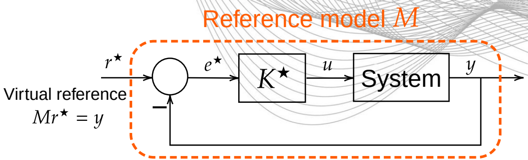
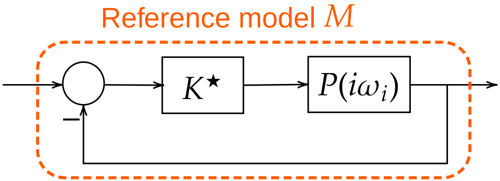


L-DDC

$$\frac{PK^\star}{1+PK^\star} = M \rightarrow K(\imath\omega_i) = K^\star(\imath\omega_i)$$



Virtual reference feedback tuning: a direct method for the design of feedback controllers, Campi et al. (2002).
Interpolation-based irrational model control design and stability analysis, Poussot-Vassal, Kergus, Vuillemin (2022).

Laplace

Discussion

- These approaches consist in **approximating the ideal controller** $K^\star$

$$\frac{PK^\star}{1+PK^\star} = M$$

Data-Driven Control: Part Two of Two: Hot Take: Why not go with Models?, F. Dörfler, *IEEE Control Systems Magazine*, 2023

Laplace

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So why not go with models?

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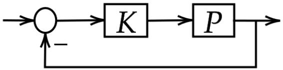
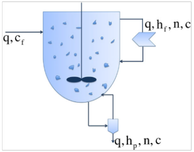
So why not go with models?

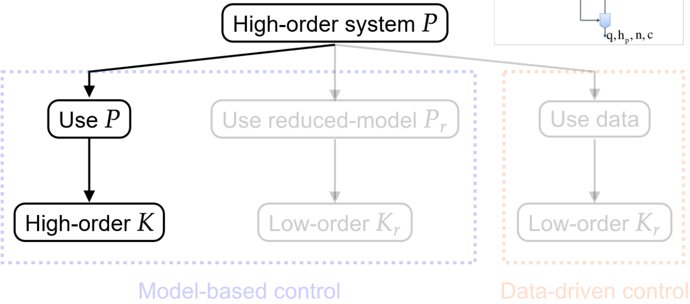
- Overcoming complexity (high dimension, non-linearities, uncertainty)

Data-Driven Control: Part Two of Two: Hot Take: Why not go with Models?, F. Dörfler, *IEEE Control Systems Magazine*, 2023

Application to infinite dimensional systems







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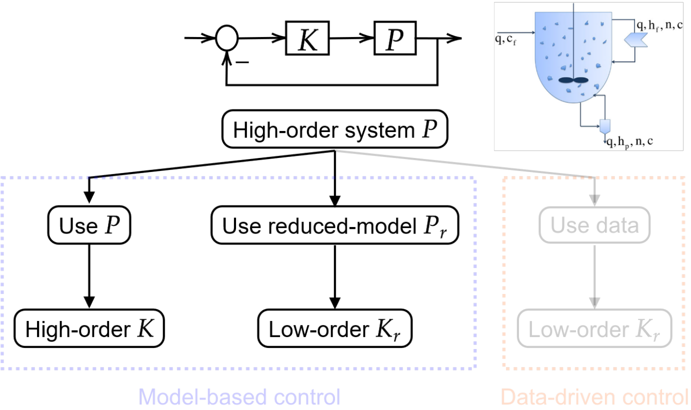

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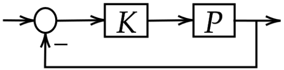

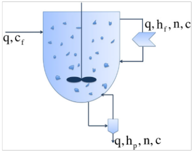

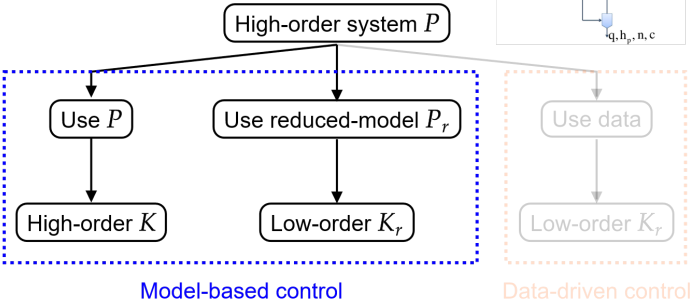

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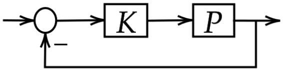

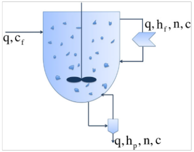

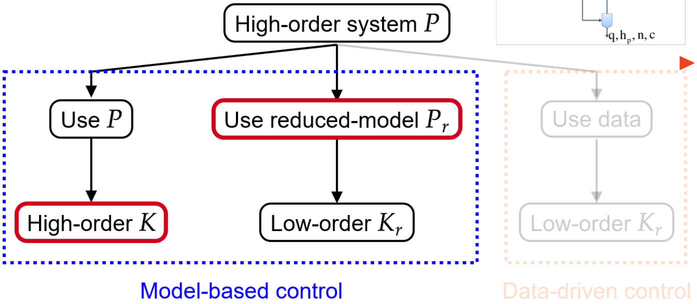

- Model Order Reduction is essential

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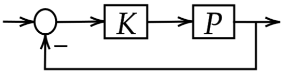

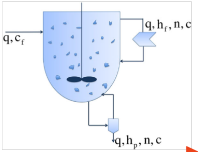

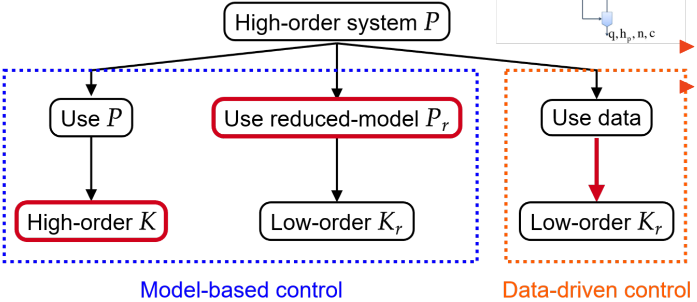

- Model Order Reduction is essential
- and it can also be used for data-driven control.

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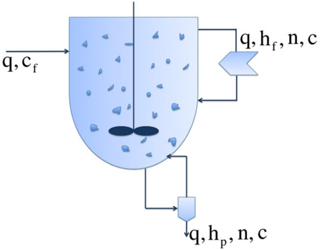
[3] *From reference model selection to controller validation: Application to Loewner Data-Driven Control*, Kergus, Olivi, Poussot-Vassal, Demourant, *IEEE Control Systems Letters*, 2019.

Laplace

Case study: the continuous crystallizer



- **Goal:** stabilize the plant around $c_{ss} = 4.09 mol/L$

1. *A mathematical model for continuous crystallization*, Rachah, Noll, Espitalier, Baillon, *Mathematical Methods in the Applied Sciences*, 2016.
2. *$\mathcal{H}_\infty$-Control of a continuous crystallizer*, Vollmer, Raisch, *Control Engineering Practice*, 2001.
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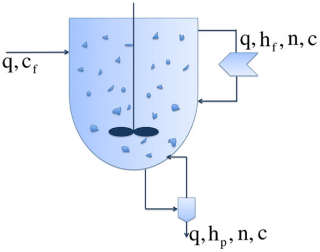

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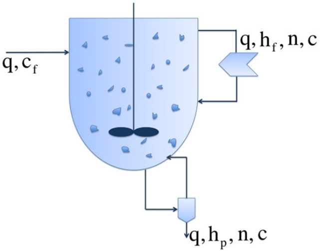

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- Linearization of the PDEs around c_{ss}

$$P(s) = \frac{\Delta c(s)}{\Delta c_f(s)} = \frac{p_{12}(s)}{p_{13}(s) + q_{12}(s)e^{-sk_f} + r_{12}(s)e^{-sk_p}}$$

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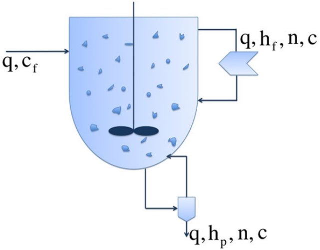

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$\rightarrow$ Frequency-domain data easily accessible
$N = 500$ frequencies, logspaced between 10^{-3} and 1 rad.s^{-1}

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L-DDC Step 1: Building a reference model

$$\left\{\begin{array}{l} \mathbf{y}_{z_i}^T \mathbf{P}(z_i) = 0 \\ \mathbf{y}_{p_j} \mathbf{P}(p_j) = \infty \end{array}\right. \quad \Rightarrow \quad \boxed{\left\{\begin{array}{l} \mathbf{y}_{z_i}{}^T \mathbf{M}(z_i) = 0 \\ \mathbf{M}(p_j)\mathbf{y}_{p_j} = \mathbf{y}_{p_j} \end{array}\right.}$$

Is the system stable or unstable? non-minimum phase or not?

Achievable performance of multivariable systems with unstable zeros and poles, Havre, Skogestad, *International Journal of Control*, 2001.

Laplace

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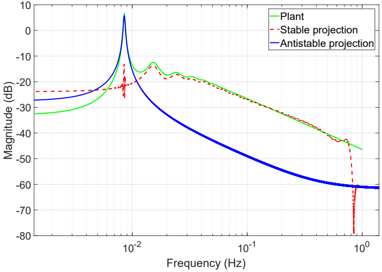

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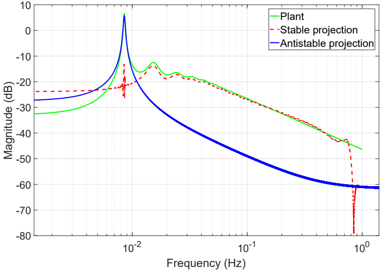

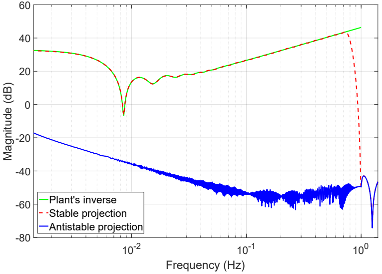

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Estimating unstable poles in simulations of microwave circuits, Cooman, Seyfert, Amari, *International Microwave Symposium*, 2018.

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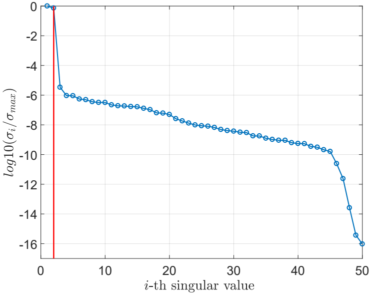
Principal Hankel Component analysis on P_{as}



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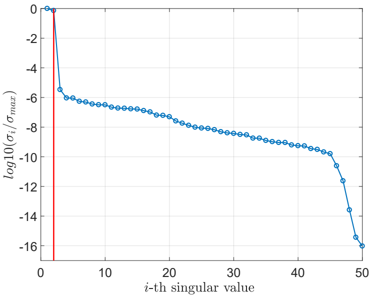

- Estimated RHP poles : $1.07 \times 10^{-4} \pm 0.852 \times 10^{-2}\jmath$
- RHP poles (direct search): $0.99 \times 10^{-4} \pm 0.89 \times 10^{-2}\jmath$

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How to choose the reference model accordingly?

$$M_{init}(s) = \frac{1}{1+\tau s}, \ \tau = 1s$$

$$\boxed{M = 1 - (1 - M_{init})B_p}$$

$$B_p(s) = \prod_{j=1}^{n_p} \frac{s-p_j}{s+p_j}$$

$$\forall j = 1 \ldots n_p, \ B_p(p_j) = 0$$

$$\forall \omega, \ |B_p(\jmath\omega)| = 1$$

Laplace

Interlude: the Loewner framework

Find $\mathbf{g}$ such that $\begin{cases} \mathbf{g}(\lambda_j) & = & \mathbf{w}_j,\ j = 1, \ldots, k \\ \mathbf{g}(\mu_i) & = & \mathbf{v}_i,\ i = 1, \ldots, q \end{cases}$

Lagrangian form

$$\mathbf{g}(s) = \frac{\sum_{j=1}^{k_1} \frac{c_j \mathbf{w}_j}{s-\lambda_j}}{\sum_{j_1=1}^{k_1} \frac{c_j}{{}^1 s-\lambda_j}}$$

A tutorial introduction to the Loewner framework for model reduction, Antoulas et al. (2017).

Interlude: the Loewner framework

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Loewner matrix

$$\mathbb{L} \in \mathbb{C}^{q \times k}$$

$$(\mathbb{L})_{i,j} = \frac{\mathbf{v}_i - \mathbf{w}_j}{\mu_i - \lambda_j}$$

Null space

$$\textbf{span}\ (\mathbf{c}) = \mathcal{N}(\mathbb{L})$$

$$\mathbf{c} = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_{k_1} \end{bmatrix} \in \mathbb{C}^{k_1}$$

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Interlude: the Loewner framework

$$\begin{cases} \mathbf{E}\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u \\ y = \mathbf{C}\mathbf{x} \end{cases}$$

$$\mathbf{H}(s) = \mathbf{C}(s\mathbf{E} - \mathbf{A})^{-1}\mathbf{B}$$

Objective: Find $\left(\hat{\mathbf{E}}, \hat{\mathbf{A}}, \hat{\mathbf{B}}, \hat{\mathbf{C}}\right)$ such that for a set of interpolation points $\{s_k\}$

$$\forall i,\ \hat{\mathbf{H}}(\lambda_i)\mathbf{r}_i = \mathbf{w}_i$$

$$\forall j,\ \mathbf{l}_j\hat{\mathbf{H}}(\mu_j) = \mathbf{v}_j$$

Interlude: the Loewner framework

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$$\boxed{\begin{cases} -\mathbb{L}\dot{\tilde{\mathbf{x}}} = -\mathbb{L}_s\tilde{\mathbf{x}} + \mathbf{V}\mathbf{u} \\ \tilde{\mathbf{y}} = \mathbf{W}\tilde{\mathbf{x}} \end{cases}}$$

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- $\mathbb{L}$, $\mathbb{L}_s$, $\mathbf{V}$ and $\mathbf{W}$ are data matrices
- Model order reduction based on SVD of the pencil $(\mathbb{L}, \mathbb{L}_s)$

$$(\mathbb{L})_{i,j} = \frac{\mathbf{v}_i\mathbf{r}_i - \mathbf{l}_j\mathbf{w}_j}{\mu_i - \lambda_j} \quad (\mathbb{L}_s)_{i,j} = \frac{\mu_i\mathbf{v}_i\mathbf{r}_i - \lambda_j\mathbf{l}_j\mathbf{w}_j}{\mu_i - \lambda_j}$$

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- Factorization in terms of the tangential generalized controllability $\mathcal{R}$ and observability $\mathcal{O}$ matrices:

$$\mathbb{L} = -\mathcal{O}\mathbf{E}\mathcal{R} \quad \mathbb{L}_s = -\mathcal{O}\mathbf{A}\mathcal{R} \quad \mathbf{V} = \mathbf{C}\mathcal{R} \quad \mathbf{W} = \mathcal{O}\mathbf{B}$$

$$\mathbf{A}\mathcal{R} + \mathbf{B}\mathbf{R} = \mathbf{E}\mathcal{R}\mathbf{\Lambda} \quad \mathcal{O}\mathbf{A} + \mathbf{L}\mathbf{C} = \mathbf{M}\mathcal{O}\mathbf{E}$$

LDDC Step 2: Controller identification and reduction

Objective: obtain a rational model $\mathbf{K} = (E, A, B, C, D)$ such that:

$$\forall i = 1 \ldots N, \mathbf{K}(\imath\omega_i) = \mathbf{K}^\star(\imath\omega_i) = \frac{M(\jmath\omega_i)}{P(\jmath\omega_i)(1 - M(\jmath\omega_i))}.$$

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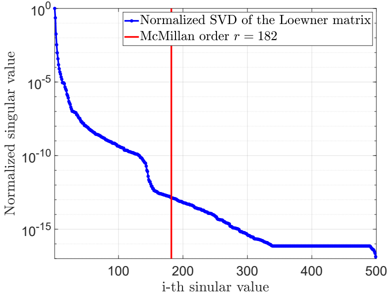


Figure: SVD of $\mathbb{L}$.

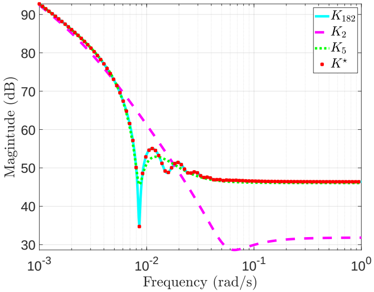
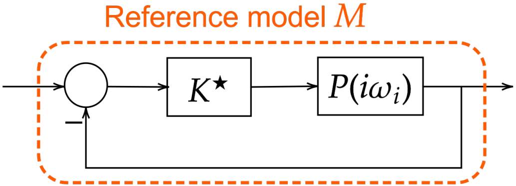


Figure: Obtained controllers.

Laplace

LDDC Step 3: Closed-loop stability analysis



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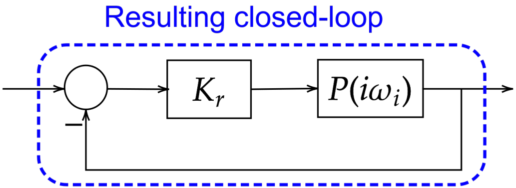

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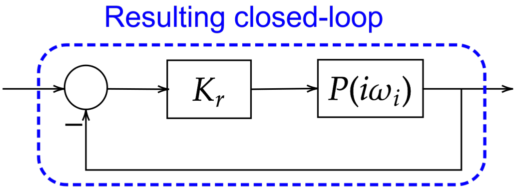

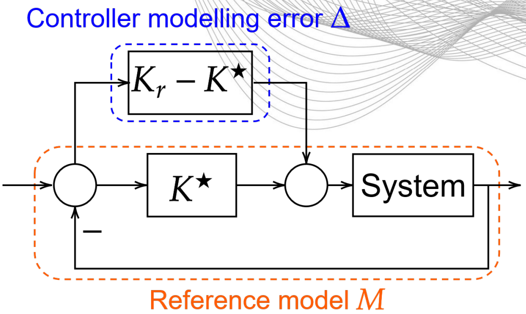

Application of the small-gain theorem

The closed-loop is well-posed and internally stable for all stable $\Delta = K - K^\star$ such that $\|\Delta\|_\infty \leq \beta$ if and only if $\|(1 - \mathbf{M})\mathbf{P}\|_\infty < \frac{1}{\beta}$

$\rightarrow$ Limiting the controller modelling error allows to ensure closed-loop internal stability!

Data-driven controller validation, Van Heusden, Karimi, Bonvin, *IFAC Proceedings*, 2009.

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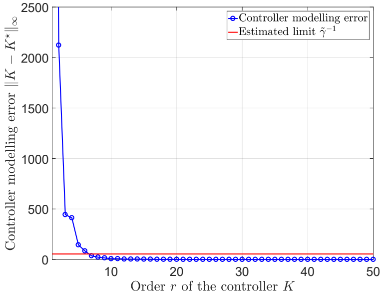

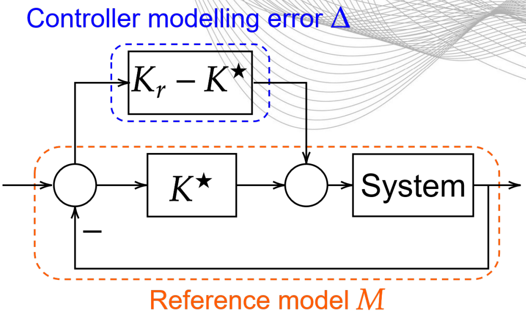

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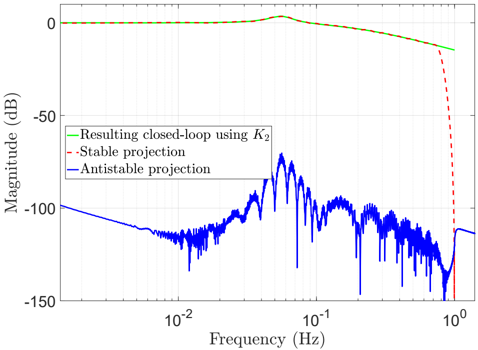
Alternative closed-loop stability analysis

$$H(\jmath\omega_i) = \frac{P(\jmath\omega_i)K_r(\jmath\omega_i)}{1 + P(\jmath\omega_i)K_r(\jmath\omega_i)}$$

1st option



Model-free closed-loop stability analysis: A linear functional approach, Cooman, Seyfert, Olivi, Chevillard, Baratchart, *IEEE Transactions on Microwave Theory and Techniques*, 2018

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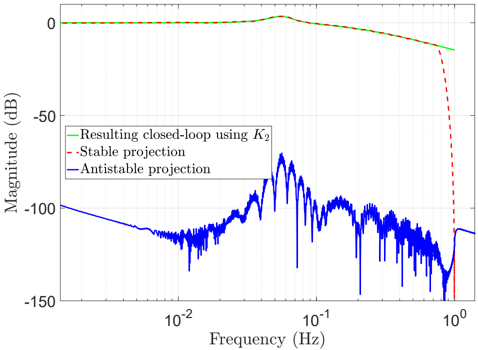

2nd option

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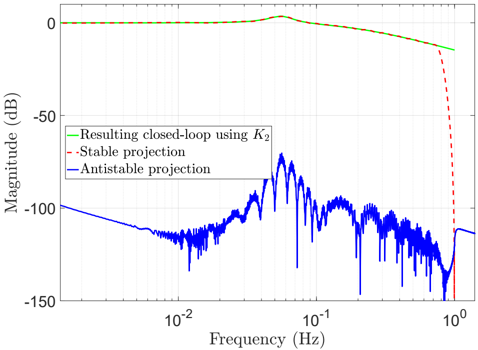

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On the closest stable descriptor system in the respective spaces $\mathcal{RH}_2$ and $\mathcal{RH}_\infty$, Köhler, *Linear Algebra and its Applications*, 2014.

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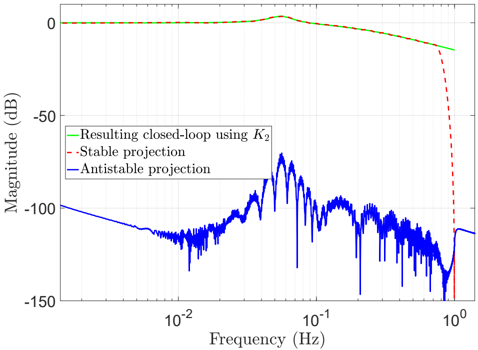

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3. Stability index $S = \|\hat{H}_s - \hat{H}\|_\infty$

$$\boxed{S = 4.3511 \cdot 10^{-6}}$$

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Laplace

Results

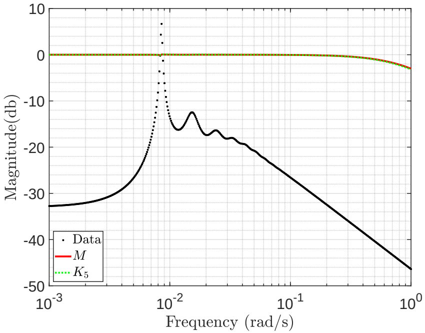


Figure: Closed-loop transfer functions.

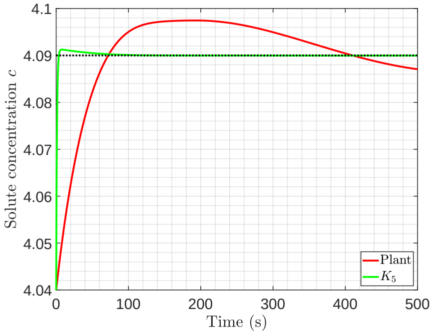
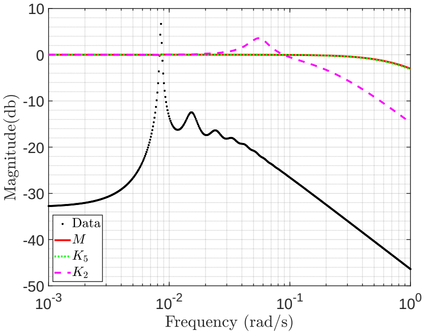


Figure: Time-domain simulation.

Results



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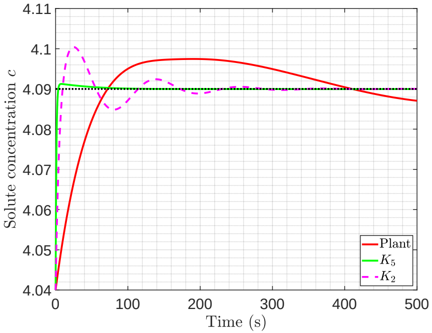

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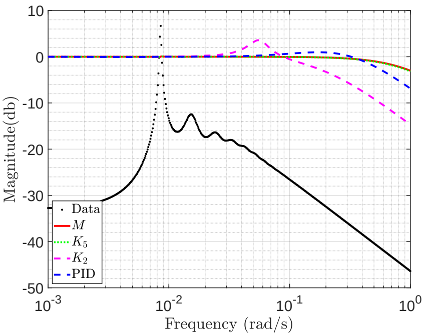
⇒ Impact of the **complexity-accuracy trade-off**

Laplace

Results



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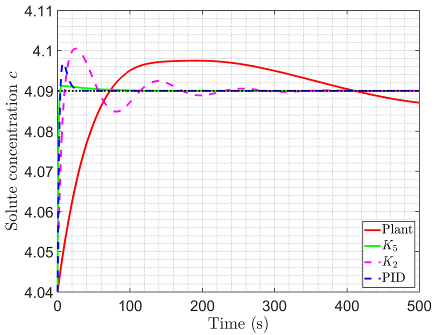

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$\Rightarrow$ Impact of the **complexity-accuracy trade-off**

$\rightarrow$ Impact of the reference model (comparison with a robust PID)

Structured H control of infinite-dimensional systems, Apkarian and Noll, 2018.

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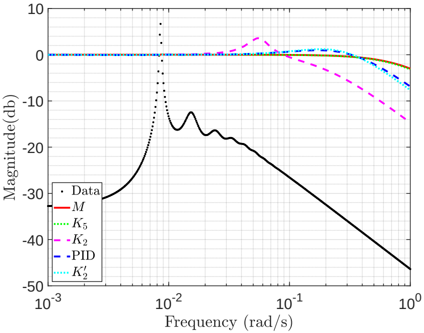

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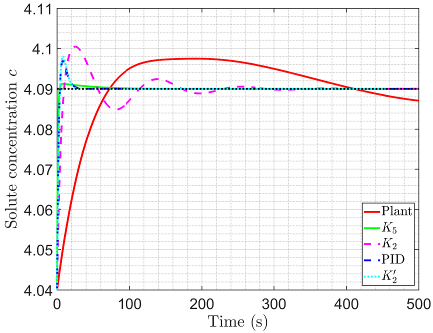

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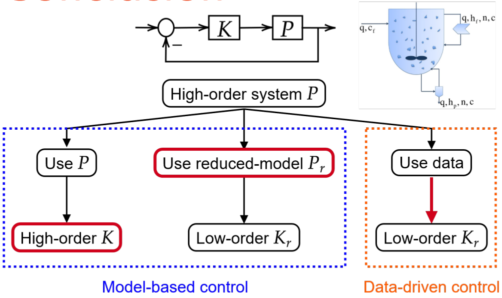
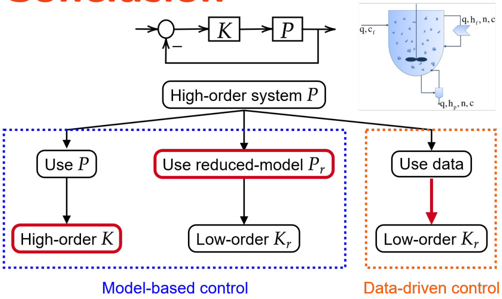
Conclusion



The Loewner framework can be used as a central tool for the control of infinite dimensional transfer functions.

Laplace

Conclusion



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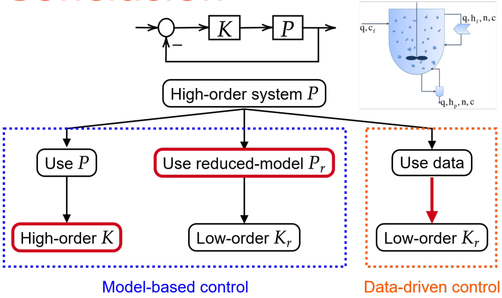
	Model-based design	L-DDC
Method	more steps	direct
Controller structure	fixed order/poles	linear
Specifications	flexible (robust)	not flexible (only stability)
Stability guarantees	for P_r	conservative or not embedded

Conclusion



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Extension to other types of systems?

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Laplace