Restricted Model Free Control : toward a geometrical framework

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Facilities at ENAC

Motivating example: Convertible drone flight

- Many modern systems are **too complex to be reliably modeled** and are also **critical systems**.
- Convertible drones: **highly coupled aerodynamics** and **multiple flight regimes**
- This motivates the need for **robust, data-driven control strategies**.

- **Incremental Nonlinear Dynamic Inversion (INDI):** Relies on *incremental measurements* rather than a full model. Robust, but still model-dependent for tuning.[4]

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[5] Fliess & Join, *Int. J. Control*, 2013.
[6] Fliess & Join, *IFAC*, 2009.

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Introduction

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- **Model-Free Control (MFC):** Introduces the *ultra-local model*. Simple, robust, but purely data-driven.[5]
- **Restricted Model-Free Control (RMC):** Extension of MFC restricted to **flat outputs**. Gives physical meaning to the ultra-local model.[6]

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- **Restricted Model-Free Control (RMC):** Extension of MFC restricted to **flat outputs**. Gives physical meaning to the ultra-local model.[6]

- **Limitations:** For systems with **symmetries** ($SE(2), SE(3)$), the standard RMC generally does not preserve them : *coordinate-dependent behavior*.

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Related Work - Control and Symmetries

- **Nonlinear methods:** Most prominent works are from the 1990s, e.g. Tracking for fully actuated mechanical systems: a geometric framework by Bullo and Murray.[7]
- **Linear methods:** More recent approaches, such as invariant tracking and stabilization (See P. Rouchon., J. Rudolph)[8], application to quadrotors [9], or exploiting different symmetries in the estimation field [10].

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[7] Bullo, F., Murray, R. M. (1990), *Automatica*.
[8] Rouchon, P., Rudolph, J. (1999), springer.
[9] Cohen & al. (2020), *IEEE Robotics and Automation Letters*.
[10] "Symmetry-preserving obervers". Bonnabel & al. *IEEE-TAC*. 2008

Plan

1. Conventional RMC and motivating example
2. Invariant RMC methodology
3. Application to unicyle

The Resticted Model-free Control framework

Nonlinear system with state $x(t)$

$$\dot{x} = f(x,u), \quad y = h(x), \quad x \in \mathbb{R}^n,\ u \in \mathbb{R}^m,\ y \in \mathbb{R}^p$$

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- **Differentially flat**, $\exists$ a flat output $z(t)$ such that :

$$x(t) = \phi_x(z, \dot{z}, \dots, z^{(r)}), \quad u(t) = \phi_u(z, \dot{z}, \dots, z^{(r)}).$$

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- **A nominal trajectory :** $(x^\star(t),\ u^\star(t)),\ y^\star(t) = h(x^\star(t))$

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Ultra local model: $e^{(\nu)}(t) = F(t) + \alpha(t)\,\delta u(t)$, $e = y - y^\star$.

Goal: design a control law $u(t) = \delta u(t) + u^\star(t)$ such that :

$$\begin{aligned} \delta u(t) \ &= \ \alpha(t)^{-1}\Big(\widehat{F}(t) - K(e(t))\Big) \\ e(t) = y(t) &- y^\star(t) \to 0 \end{aligned}$$

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What about this "linear" error when the state space is a manifold, a Lie group?

Motivating example : Classical v.s invariant

The unicyle motion ...

$$\dot{x}_p = v\cos\theta,\ \ \dot{y}_p = v\sin\theta,\ \ \dot{\theta} = \omega.$$

... is flat with flat output $\begin{bmatrix} x_p & y_p \end{bmatrix}^T$.

We can compute the corresponding nominal orientation $\theta^\star$ and nominal control input $U^\star = [v^\star, \omega^\star]^T$ with :

$$v^\star = \sqrt{\dot{x}_p^{\star\ 2} + \dot{y}_p^{\star\ 2}}$$

$$\omega^\star = \frac{\ddot{y}_p^\star \dot{x}_p^\star - \ddot{x}_p^\star \dot{y}_p^\star}{\dot{x}_p^{\star\ 2} + \dot{y}_p^{\star\ 2}}$$

And the linear error :

$$e = \begin{bmatrix} e_x \\ e_y \end{bmatrix} = \begin{bmatrix} x_p - x_p^\star \\ y_p - y_p^\star \end{bmatrix}$$

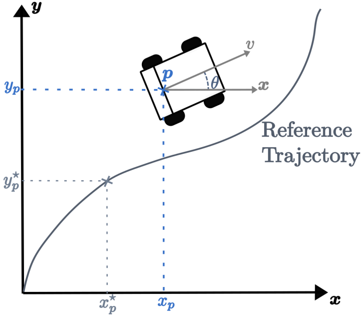


Figure: The unicyle motion in the 2D plan

Motivating example : Classical v.s invariant

Ultra-local model (relative degree) :

$$\begin{bmatrix} \dot{e}_x \\ \ddot{e}_y \end{bmatrix} = \begin{bmatrix} F_x \\ F_y \end{bmatrix} + \boldsymbol{\alpha} \begin{bmatrix} \delta v \\ \delta \omega \end{bmatrix} \tag{1}$$

$\boldsymbol{\alpha}$ matrix, given by HEOL settings[7] based on $\boldsymbol{Y}^\star$ dynamics:

$$\boldsymbol{\alpha} = \begin{bmatrix} \cos(\theta^\star) & 0 \\ \omega^\star \cos(\theta^\star) & v^\star \cos(\theta^\star) \end{bmatrix} \tag{2}$$

MFC feedback computation, with $\hat{\boldsymbol{F}}$ an estimate of $\boldsymbol{F}$:

$$\boldsymbol{\delta U} = \boldsymbol{\alpha}^{-1} \left(-\hat{\boldsymbol{F}} + \begin{bmatrix} K_{px}\, ex \\ K_{py}\, ey + K_{dy}\, \dot{e}_y \end{bmatrix} \right) \tag{3}$$

$\boldsymbol{\alpha}$ can not be inverted with $\mathbf{cos(\theta^\star) = 0}$

[4] C., Delaleau, E., and Fliess, M. (2024). Flatness-based control revisited: The HEOL setting. *Comptes Rendus. Mathématique*, 362(G12), 1693–1706.

Motivating example : Classical v.s invariant

The nonholonomic car ...

$$\dot{x}_p = v\cos\theta,\ \dot{y}_p = v\sin\theta,\ \dot{\theta} = \omega.$$

... is invariant with respect to $SE(2)$ the Lie group of rigid body motions in the plane $\Rightarrow$ use a Frenet Frame attached to reference trajectory:

$$\tau = \begin{bmatrix}\cos\theta^\star \\ \sin\theta^\star\end{bmatrix}, \quad \nu = \begin{bmatrix}-\sin\theta^\star \\ \cos\theta^\star\end{bmatrix}.$$

New error (Frenet frame):

$$e = R(\theta^\star)^\top\big((x_p, y_p) - (x_p^\star, y_p^\star)\big).$$

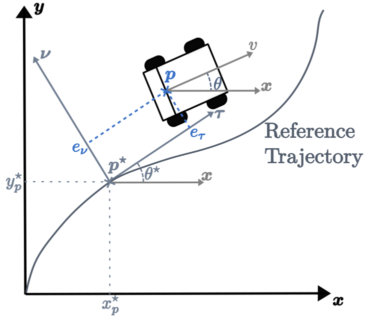


Figure: Unicyle motion and the local frame in the 2D plan

Motivating example : Classical v.s invariant

Adaptation of HEOL settings to use the dynamics of the new error:

$$
\begin{aligned}
\dot{e}_\tau &= v\cos(\theta-\theta^\star) - v^\star + \omega^\star e_\nu \\
\ddot{e}_\nu &= v\omega\cos(\theta-\theta^\star) - v^\star\omega^\star + \dot{v}\sin\theta-\theta^\star \\
&\quad - 2\omega^\star\left(v\cos(\theta-\theta^\star) - v^\star\right) - \dot{\omega}^\star e_\tau - \omega^{\star 2} e_\nu
\end{aligned}
$$

The new ultra-local model is given by:

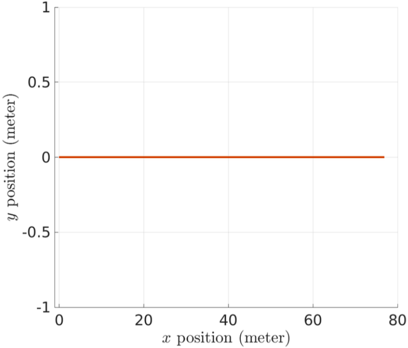
$$
\begin{bmatrix}\dot{e}_\tau \\ \ddot{e}_\nu\end{bmatrix} = \begin{bmatrix}F_\tau \\ F_\nu\end{bmatrix} + \boldsymbol{\alpha_{inv}} \begin{bmatrix}\delta v \\ \delta\omega\end{bmatrix}
$$

with:

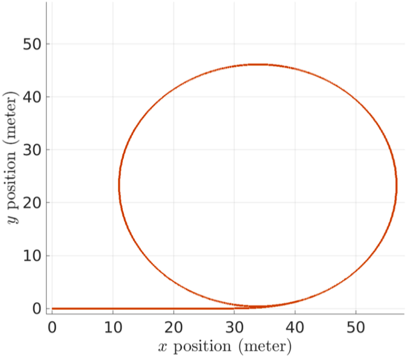
$$
\boldsymbol{\alpha_{inv}} = \begin{bmatrix}1 & 0 \\ -\omega^\star & v^\star\end{bmatrix}
$$

"α" depends only on u^*!

Simulation Results

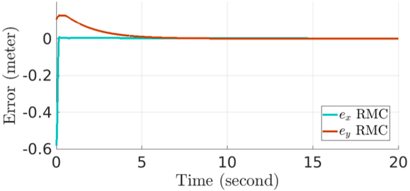


Straight line trajectory

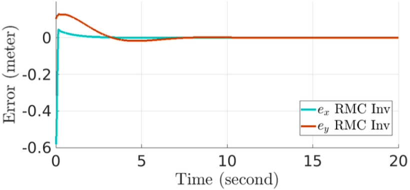


Circular trajectory

Trajectory errors



Straight line, classical RMC



Straight line, invariant RMC

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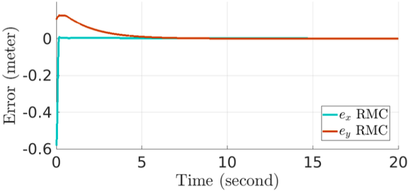

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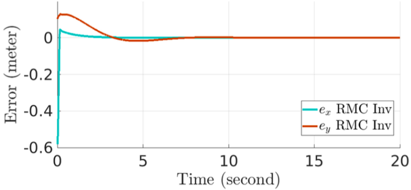

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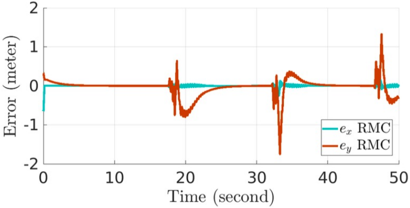
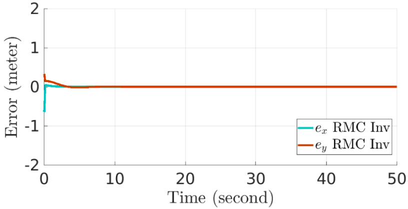


Circular trajectory, classical RMC



Circular trajectory, invariant RMC

Introduction - Lie Groups

A Lie group is a set G such that:

- $\forall \chi \in G, \exists \chi^{-1} \in G$ such that $\chi^{-1} \circ \chi = \chi \circ \chi^{-1} = I$
- $\forall \chi_1, \chi_2 \in G,\ \chi_1 \circ \chi_2 \in G$
- $\forall \chi_1, \chi_2, \chi_3 \in G,\ \chi_1 \circ (\chi_2 \circ \chi_3) = (\chi_1 \circ \chi_2) \circ \chi_3$

Ex: Matrix Lie groups:

- $SO(2)$: orientation in the 2D plane
- $SE(2)$: orientation and position in the 2D plane
- $SO(3)$: attitude in 3D space
- $SE(3)$: attitude and position in 3D space

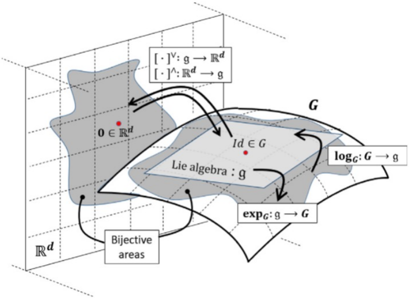
Introduction to Lie Algebra

The Lie algebra of a Lie group G is defined as the tangent space at $(\mathfrak{g} = T_I G)$. It is a **vector space**, and it is isomorphic to $\mathbb{R}^d$.

The mapping:
wedge$[\cdot]^\wedge : \mathbb{R}^d \mapsto \mathfrak{g}$, vee$[\cdot]^V : \mathfrak{g} \mapsto \mathbb{R}^d$

provides the correspondence between the Lie algebra and the Euclidean space.

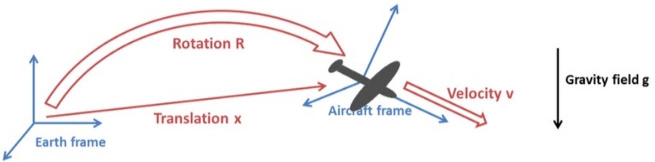
Ex: Lie group $\chi \in SE(2)$ and its Lie algebra $\mathfrak{se}(2)$:



$$\chi = \begin{bmatrix} R(\theta) & p \\ 0 & 1 \end{bmatrix} \in SE(2), \qquad \xi^\wedge = \log(\chi) = \begin{bmatrix} 0 & -\theta & \rho_1 \\ \theta & 0 & \rho_2 \\ 0 & 0 & 0 \end{bmatrix} \in \mathfrak{se}(2).$$

The Lie algebra can be seen as a linearization of the Lie group around the identity of the group I

Dynamics on matrix Lie groups



General form : For a system with configuration $\chi \in G$,

$$\dot{\chi} = f_u(\chi), \quad \text{with input} \quad u \in \mathbb{R}^p.$$

Example: Quadrotor on $SE(3)$

$$\dot{R} = R\,\omega^\times, \quad \dot{v} = g + Ru, \quad \dot{x} = v,$$

attitude $R \in SO(3)$, velocity $v \in \mathbb{R}^3$, position $x \in \mathbb{R}^3$, input $u \in \mathbb{R}^3$.

$$\chi = \begin{pmatrix} R(\theta) & v & x \\ 0_{1,3} & 1 & 0 \\ 0_{1,3} & 0 & 1 \end{pmatrix}, \log(\chi) = \begin{pmatrix} (\xi)_\times & u & v \\ 0_{1,3} & 0 & 0 \\ 0_{1,3} & 0 & 0 \end{pmatrix} \in \mathfrak{se}(3), \quad \dot{\chi} = f_u(\chi).$$

Ingredients of Invariant RMC on Lie Groups

Invariant error: Given true trajectory χ and reference χ_d (or $\chi^\star$)

$$\eta = \chi_d^{-1}\chi \in G, \qquad \eta = I \;\; \Leftrightarrow \;\; \chi = \chi_d$$

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Group-affine dynamics (Bonnabel & al, 2017):
if

$$f_u(gh) = f_u(g)h + gf_u(h) - gf_u(I)h,$$

then the error is *autonomous*:

$$\dot{\eta} = f_u(\eta) - f_u(I)\,\eta.$$

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Local error dynamics: in logarithmic coordinates $\varepsilon = \log(\eta)$,

$$\dot{\varepsilon} = A(t)\,\varepsilon + B(t)\,\tilde{u} + r(\varepsilon, \tilde{u}, t),$$

with $\tilde{u} = u - u^\star$, and $r = O(\|\varepsilon\|^2 + \|\varepsilon\|\|\tilde{u}\|)$.

- $A(t), B(t)$ depend only on reference $(x^\star, u^\star)$
- **Equivariant linearization:** structure-preserving, remainder is higher order $\Rightarrow$ locally like a linear control system in ε

Back to the example: Invariant RMC

Step 1: Small Error Dynamics: For small errors $(\varepsilon_\tau, \varepsilon_\nu, (\theta - \theta_d))$, the dynamics are given by:

$$\dot{\varepsilon} \;\simeq\; A(t)\,\varepsilon + B(t)\,\delta u.$$

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Step 2: Error in $\mathfrak{se}(2)$: The error dynamics in $\mathfrak{se}(2)$:

$$\xi = (\varepsilon_\tau, \varepsilon_\nu, (\theta - \theta_d)), \quad \dot{\xi} = \mathrm{ad}_{\upsilon_d}(\xi) + (\upsilon - \upsilon_d).$$

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Step 3: Adjoint Action: Adjoint action of υ_d (from $SE(2)$):

$$\mathrm{ad}_{\upsilon_d} = \begin{bmatrix} 0 & -\omega^\star & 0 \\ \omega^\star & 0 & -v^\star \\ 0 & 0 & 0 \end{bmatrix}.$$

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Step 4: Outputs and Relative Degree: Outputs $(\varepsilon_\tau, \dot{\varepsilon}_\nu)$, relative degree:

$$\begin{bmatrix} \dot{\varepsilon}_\tau \\ \ddot{\varepsilon}_\nu \end{bmatrix} = \begin{bmatrix} F_\tau \\ F_\nu \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ -\omega^\star & v^\star \end{bmatrix} \begin{bmatrix} \delta v \\ \delta\omega \end{bmatrix}.$$

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$\Rightarrow$ Projection recovers Frenet dynamics, linking error and control in $SE(2)$.

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Ultra local model: $\varepsilon^{(\nu)}(t) = F(t) + \alpha(t)\,\delta u(t)$, $\varepsilon = \log(\chi_d^{-1}\chi)^V$

Goal: design a control law $u(t) = \delta u(t) + u_d(t)$ such that :

$$\delta u(t) \;=\; \alpha(t)^{-1}\Big(\widehat{F}(t) - K(\varepsilon(t))\Big)$$
$$\varepsilon = \log(\chi_d^{-1}\chi)^V \to 0$$

Conclusion

Summary:

- We extended classical RMC with an invariant framework for systems on Lie groups.
- This method preserves symmetries and provides robustness for systems with complex or uncertain models (e.g., quadrotors).
- The error dynamics in Lie groups allow for convergence over larger regions than classical methods, improving stability.

Future Work:

- Extend the framework to other invariant ODEs (e.g., using the infinitesimal generator).
- Refine stability guarantees, including uncertainty estimation.