A hybrid systems framework for data-based adaptive control of linear time-varying systems

Andrea Iannelli

Institute for Systems Theory and Automatic Control, University of Stuttgart - Germany

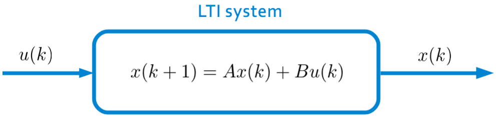
Romain Postoyan

CNRS, Université de Lorraine, CRAN - France

Data-driven control inspired by Willems' fundamental lemma

e.g., [Willems et al. SCL'05, de Persis & Tesi IEEE TAC'20, van Waarde et al. book'25]

1. LTI system with unknown matrices $A \in \mathbb{R}^{n_x \times n_x}$ and $B \in \mathbb{R}^{n_x \times n_u}$ with $n_x, n_u \in \mathbb{Z}_{>0}$



2. Collect data: $[u(0), u(1), \ldots, u(T-1)]$ and $[x(0), x(1), \ldots, x(T)]$ with $T \in \mathbb{Z}_{\geq 1}$

3. Derive the class of systems/matrices Σ that explains the data

4. Design a controller that achieves the desired property for any system in Σ

Pros

- Direct control based on data
- No need to identify the system matrices

Features

- Data collected off-line
- Off-line design

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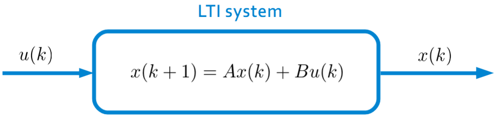

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From LTI to LTV

LTV system

$$u(k) \longrightarrow \boxed{x(k+1) = A(k)x(k) + B(k)u(k)} \longrightarrow x(k)$$

with $A(k) \in \mathbb{R}^{n_x \times n_x}$ and $B(k) \in \mathbb{R}^{n_x \times n_u}$ with $n_x, n_u \in \mathbb{Z}_{>0}$

Questions

When and how to learn a new feedback law?

Slowly varying systems

[Liu et al. IEEE CDC'23]

- Periodic control updates
- Control designed on-line

Switched systems

e.g., [Eising et al. TAC'24]

- Event-based control updates
- Control designed off-line

Contributions

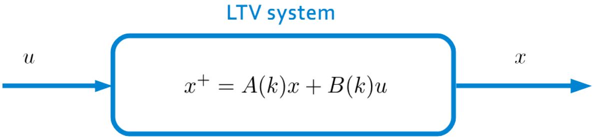
Event-based learning like in [Eising et al. IEEE TAC'24]

Control gain design on-line

No explicit restrictions on the variations of A and B

General conditions for various stability properties $\rightsquigarrow$ may be difficult to check
$\rightarrow$ first case studies

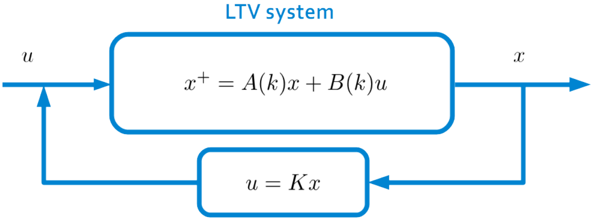
Set-up



Objective

To stabilize $x = 0$

Set-up



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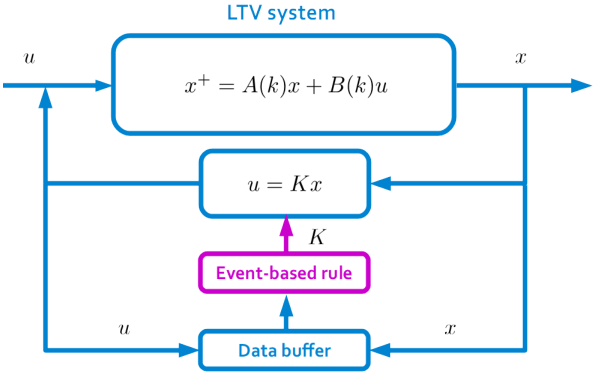

Objective

To stabilize $x = 0$

Closed-loop modeling

Hybrid discrete-time model

As in [Sanfelice & Teel Automatica'10]

$$\begin{cases} q^+ = f(q) & q \in \mathcal{C} \\ q^+ \in G(q) & q \in \mathcal{D}, \end{cases}$$

$q = \begin{pmatrix} x \\ K \\ \text{data} \\ \dots \end{pmatrix}$: state $\mathcal{C}$: system operation $\mathcal{D}$: episode triggering

Solution parameterized by two discrete times $q(k, j)$:

- $k \in \mathbb{Z}_{\geq 0}$ physical time
- $j \in \mathbb{Z}_{\geq 0}$ number of episodes

similar concepts as in [Goebel et al. Princeton Univ. Press'12]

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LTV system

Discrete-time LTV system

$$x^{+} = A(k)x + B(k)u,$$

with $x \in \mathbb{R}^{n_x}$, $u \in \mathbb{R}^{n_u}$

Counter variable κ:

$$\begin{aligned} \kappa^{+} &= \kappa + 1 && \text{(system operation)} \\ \kappa^{+} &= \kappa && \text{(episode triggering)} \end{aligned}$$

Hence

$$x^{+} = A(\kappa)x + B(\kappa)u$$

Data variables

Let $T \in \mathbb{Z}_{>0}$.

When the system is operated

- $\widehat{X}$: stacking of the last T values of x

$$\widehat{X}(0) = [\widehat{X}_1(0), \widehat{X}_2(0), \dots, \widehat{X}_{T-1}(0), \widehat{X}_T(0)] \in \mathbb{R}^{n_x \times T}$$

- X: stacking of the last T values of x^+

$$X^+ = [X_{2:T}, A(\kappa)x + B(\kappa)u]$$

- U: stacking of the last T values of u

$$U^+ = [U_{2:T}, u]$$

At episode triggering

$$(\widehat{X}^+, X^+, U^+) = (\widehat{X}, X, U)$$

Data variables

Let $T \in \mathbb{Z}_{>0}$.

When the system is operated

- $\widehat{X}$: stacking of the last T values of x

$$\widehat{X}(1) = [\widehat{X}_2(0), \widehat{X}_3(0), \ldots, \widehat{X}_T(0), x(0)] \in \mathbb{R}^{n_x \times T}$$

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- U: stacking of the last T values of u

$$U^+ = [U_{2:T}, u]$$

At episode triggering

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Data variables

Let $T \in \mathbb{Z}_{>0}$.

When the system is operated

- $\widehat{X}$: stacking of the last T values of x

$$\widehat{X}(2) = [\widehat{X}_3(0), \widehat{X}_4(0), \ldots, x(0), x(1)] \in \mathbb{R}^{n_x \times T}$$

- X: stacking of the last T values of x^+

$$X^+ = [X_{2:T}, A(\kappa)x + B(\kappa)u]$$

- U: stacking of the last T values of u

$$U^+ = [U_{2:T}, u]$$

At episode triggering

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Data variables

Let $T \in \mathbb{Z}_{>0}$.

When the system is operated

- $\widehat{X}$: stacking of the last T values of x

$$\widehat{X}(T) = [x(0), x(1), \ldots, x(T-2), x(T-1)] \in \mathbb{R}^{n_x \times T}$$

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Data variables

Let $T \in \mathbb{Z}_{>0}$.

When the system is operated

- $\widehat{X}$: stacking of the last T values of x

$$\widehat{X}(k) = [x(k-T), x(k-T+1), \dots, x(k-2), x(k-1)] \in \mathbb{R}^{n_x \times T} \qquad k > T$$

- X: stacking of the last T values of x^+

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Controller variables

Feedback law

$$u = Kx$$

with K given by LMI involving additional variables

- S Lyapunov matrix, $V(x, S) = x^\top Sx$
- $a_1 \in [0, 1)$ "nominal decay rate" of V
- matrix F quantifies the robustness of K
- $a_2 \in \mathbb{R}_{\geq 0}$ quantifies the effect of matrix variations

When the system is operated

$$(K^+, S^+, F^+, a_1^+, a_2^+) \quad = \quad (K, S, F, a_1, a_2).$$

At episode triggering

$$(K^+, S^+, F^+, a_1^+, a_2^+) \quad \in \quad L(\widehat{X}, X, U),$$

with L set-valued map to be designed

Episode-triggering variables

Auxiliary variable $\eta \in \mathcal{S}_\eta$

When the system is operated

$$\eta^+ = h(x, \kappa, \eta)$$

At episode triggering

$$\eta^+ = \ell(x, \kappa, \eta).$$

Hybrid model & Goal

State vector

$$q := (x, \kappa, \underbrace{\widehat{X}, X, U}_{\text{data}}, \underbrace{K, S, F, a_1, a_2}_{\text{controller related}}, \eta) \in \mathcal{S}_q$$

$$q^+ = f(q) := \begin{pmatrix} A(\kappa)x + B(\kappa)Kx \\ \kappa + 1 \\ [\widehat{X}_{2:T}, x] \\ [X_{2:T}, A(\kappa)x + B(\kappa)Kx] \\ [U_{2:T}, Kx] \\ K \\ S \\ F \\ a_1 \\ a_2 \\ h(q) \end{pmatrix} \quad q \in \mathcal{C}, \qquad q^+ \in G(q) := \begin{pmatrix} x \\ \kappa \\ \widehat{X} \\ X \\ U \\ L(\widehat{X}, X, U) \\ \ell(q) \end{pmatrix} \quad q \in \mathcal{D},$$

Goal

Design

- L (how to update the control gain)
- h, ℓ, $\mathcal{C}$ and $\mathcal{D}$ (when to do so)

to stabilize under some conditions

$$\mathcal{A} := \{q \,:\, x = 0\}$$

Episode-triggering control design

Need a measure of how "outdated" are the last used data

Last used data for design

$$\mathrm{d} := (\kappa, \widehat{X}, X, U) \in \mathcal{S}_{\mathrm{d}}$$

For $\kappa > T$,

$$[x(\kappa - T + 1), \ldots, x(\kappa)] = \Big[A(\kappa - T)x(\kappa - T), \ldots, A(\kappa - 1)x(\kappa - 1)\Big] + \Big[B(\kappa - T)u(\kappa - T), \ldots, B(\kappa - 1)u(\kappa - 1)\Big]$$

For $\kappa' \geq \kappa$, consider

$$\Big[A(\kappa')x(\kappa - T), \ldots, A(\kappa')x(\kappa - 1)\Big] + \Big[B(\kappa')u(\kappa - T), \ldots, B(\kappa')u(\kappa - 1)\Big]$$

Take the difference

$$\begin{aligned}\widetilde{D}(\mathrm{d}, A(\kappa'), B(\kappa')) := & \Big[\big(A(\kappa') - A(\kappa - T)\big)x(\kappa - T), \ldots, \big(A(\kappa') - A(\kappa - 1)\big)x(\kappa - 1)\Big] \\ & + \Big[\big(B(\kappa') - B(\kappa - T)\big)u(\kappa - T), \ldots, \big(B(\kappa') - B(\kappa - 1)\big)u(\kappa - 1)\Big]\end{aligned}$$

Given some matrix $\mathbf{F}$

$$\mathcal{E}(\mathrm{d}, \mathbf{F}) := \left\{ [M_A \; M_B]^\top \; : \; \widetilde{D}(\mathrm{d}, M_A, M_B)\widetilde{D}(\mathrm{d}, M_A, M_B)^\top \preceq \mathbf{F} \right\}$$

Design K to stabilize any plant with matrices $[M_A \; M_B]^\top \in \mathcal{E}(\mathrm{d}, \mathbf{F})$

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Controller design

Feedback law

$$u = Kx$$

with K given by LMI involving additional variables

- S Lyapunov matrix, $V(x, S) = x^\top S x$
- scalar $a_1 \in [0, 1)$ "nominal decay rate" of V
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We actually need more, for all $\varepsilon \geq 0$,

$$[M_A\ \ M_B]^\top \in \mathcal{E}(\mathrm{d}, F + \varepsilon S^{-1}) \Rightarrow V((M_A + M_B K)x, S) \leq (a_1 + a_2\varepsilon) V(x, S) \quad \forall x \in \mathbb{R}^{n_x} \tag{$\star$}$$

Data-based LMI conditions inspired by[a] [de Persis & Tesi IEEE TAC'20] for L such that $(\star)$ holds

$$(K^+, S^+, F^+, a_1^+, a_2^+) \quad \in \quad L(\widehat{X}, X, U)$$

[a] Other designs possible.

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Episode triggering

When system operated, monitor whether

$$\text{“}V^+ \leq \sigma a_1 V\text{”}$$

with σ such that $\sigma a_1 \in (0, 1)$

State at the last physical time $\hat{x}$

$$\hat{x}^+ = x \quad \text{(system operation)}, \qquad \hat{x}^+ = \hat{x} \quad \text{(episode triggering)}$$

we can write

$$V(x, S) \leq \sigma a_1 V(\hat{x}, S)$$

Toggle variable τ

$$\tau^+ = 1 \quad \text{(system operation)}, \qquad \tau^+ = 0 \quad \text{(episode triggering)}$$

Sets $\mathcal{C}$ and $\mathcal{D}$

$$\begin{aligned} \mathcal{C} &:= \left\{ q : V(x,S) \leq \sigma a_1 V(\hat{x}, S) \text{ or } L(\widehat{X}, X, U) = \emptyset \text{ or } \tau = 0 \right\} && \text{(system operation)} \\ \mathcal{D} &:= \left\{ q : V(x,S) \geq \sigma a_1 V(\hat{x}, S) \text{ and } L(\widehat{X}, X, U) \neq \emptyset \text{ and } \tau = 1 \right\} && \text{(episode)} \end{aligned}$$

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Stability guarantees

Lyapunov-based analysis

Recall

$$\mathcal{C} \quad := \quad \left\{ q : V(x,S) \leq \sigma a_1 V(\hat{x},S) \text{ or } L(\widehat{X},X,U) = \emptyset \right\}$$

Define

$$\mathcal{C}_1 \quad := \quad \left\{ q : V(x,S) \leq \sigma a_1 V(\hat{x},S) \right\}$$

Lyapunov analysis with $V(x,S) = x^\top S x$

- $\lambda_{\min}(S)|x|^2 \leq V(x,S) \leq \lambda_{\max}(S)|x|^2$
- On $\mathcal{C}$,

$$V(x^+,S^+) \leq \nu_{\mathcal{C}}(q) V(x,S)$$

  with

$$\nu_{\mathcal{C}}(q) \quad = \quad \begin{cases} \sigma a_1 & \text{if } x^+ \in \mathcal{C}_1 \\ a_1 + a_2\varepsilon & \text{else} \end{cases}$$

  $\varepsilon > 0$ such that $[A(\kappa) \;\; B(\kappa)]^\top \in \mathcal{E}(\mathrm{d}, F + \varepsilon S^{-1})$

- On $\mathcal{D}$,

$$V(x^+,S^+) \leq \nu_{\mathcal{D}}(q) V(x,S)$$

General conditions

Upper bound on V along any solution q

$$V(x(k,j),S(k,j)) \leq \pi(q,k,j) V(x(0,0),S(0,0)) \qquad \forall (k,j) \in \operatorname{dom} q$$

with

$$\pi(q,k,j) := \prod_{j'=0}^{j-1} \nu_d(q_2(k_{j'+1},j')) \prod_{k'=k_{j'}}^{k_{j'+1}-1} \nu_c(q(k',j')) \prod_{k''=k_j}^{k-1} \nu_c(q(k'',j))$$

General, interpretable conditions on $\pi(q,k,j)$ and S to ensure properties for $\mathcal{A} = \{q : x = 0\}$ (stability, uniform stability, GAS, UGAS, UGES)

Notable case studies

Recall

$$\mathcal{C}_1 := \{q : V(x,S) \le \sigma a_1 V(\hat{x},S)\}$$

Denote

$$q = (x, q_2) \quad \text{with} \quad q_2 \in \mathcal{S}_2$$

Solutions eventually always in $\mathcal{C}_1$

$\exists T^\star : \mathcal{S}_2 \to \mathbb{Z}_{\ge 0}$ and $\bar{m} \in \mathbb{R}_{\ge 0}$ s.t. for any q and $(k,j) \in \operatorname{dom} q$

- For $k \ge T^\star$, $[A(\kappa(k,j)) \;\; B(\kappa(k,j))]^\top \in \mathcal{E}(\mathrm{d}(T^\star, j_{T^\star}), F(T^\star, j_{T^\star}))$
- $\max_{k \le T^\star}\{\|A(\kappa(k,j))\|, \|B(\kappa(k,j))\|, \|K(k,j)\|\} \le \overline{m}$.

Then $\mathcal{A}$ is GAS.

Furthermore, if

- $T^\star$ constant and $a_1(T^\star, j_T^\star) \le c \in [0,1)$ with c independent of solution, $\mathcal{A}$ is UGES.

Conditions for solutions frequently enough in $\mathcal{C}_1$ [Iannelli & Postoyan IEEE TAC'25], similar to Proposition 3.29 in [Goebel et al. Princeton Univ. Press'12]

Simulation results

Switching plant

Fixed matrix A

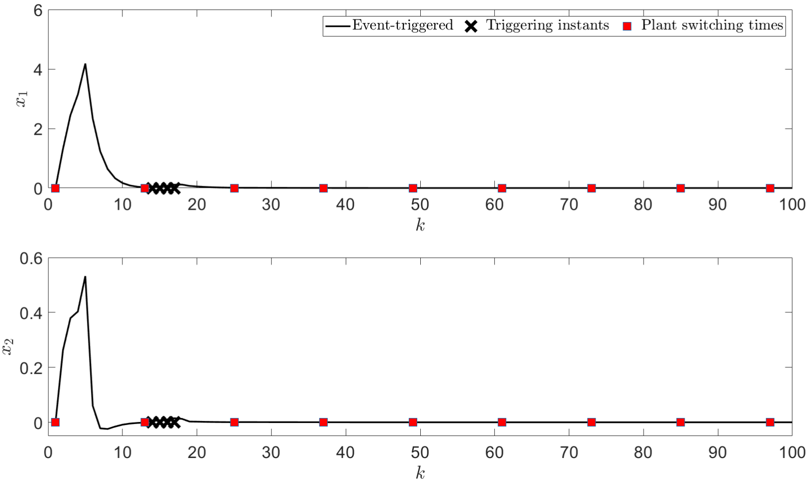
$$A(k) = \begin{bmatrix} 1.1 & 0.1 \\ 0.1 & 0.2 \end{bmatrix}, \ \forall k \in \mathbb{Z}_{\geq 0},$$

Matrix B switches every 12 steps

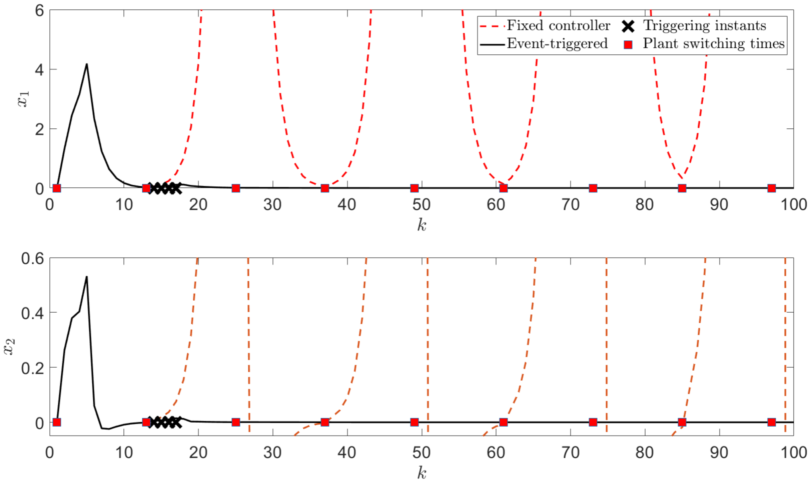
$$B(k) \in \left\{ \begin{bmatrix} 0.5 & 1 \\ 0.1 & 0.2 \end{bmatrix}, \begin{bmatrix} 0.5 & -2.5 \\ 0.1 & -0.5 \end{bmatrix} \right\}$$

- $T = n_x + n_u = 4$

Closed-loop state response



Closed-loop state response



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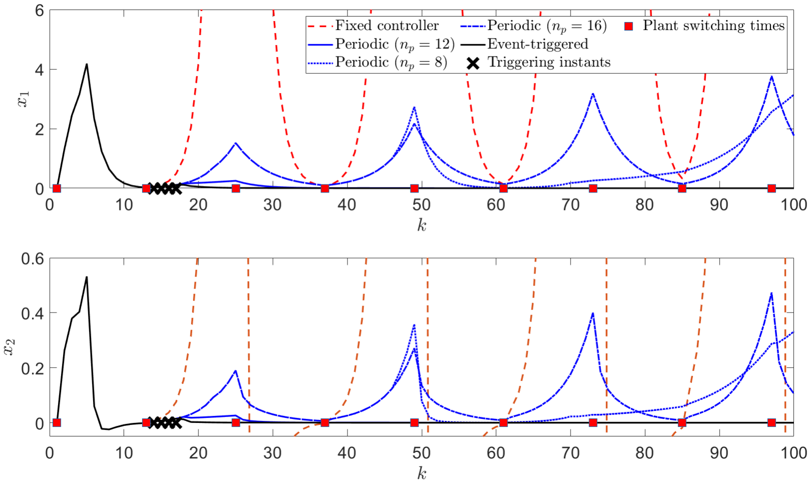

Perspectives

Perspectives

- Case studies: switched systems, classes of nonlinear systems
- Design of T (amount of collected data)
- How to enforce the feasibility of L?
- Noisy data

More details and extra results in [Iannelli & Postoyan IEEE TAC'25]

Episode triggering

When system operated, monitor whether

$$\text{“}V^+ \leq \sigma a_1 V\text{”}$$

with σ such that $\sigma a_1 \in (0,1)$

Auxiliary variables

- State at the last physical time $\hat{x}$

$$\hat{x}^+ = x \quad \text{(system operation)}, \qquad \hat{x}^+ = \hat{x} \quad \text{(episode triggering)}$$

we can write

$$V(x,S) \leq \sigma a_1 V(\hat{x},S)$$

- Toggle variable τ

$$\tau^+ = 1 \quad \text{(system operation)}, \qquad \tau^+ = 0 \quad \text{(episode triggering)}$$

Sets $\mathcal{C}$ and $\mathcal{D}$

$$\begin{aligned} \mathcal{C} &:= \left\{q : V(x,S) \leq \sigma a_1 V(\hat{x},S) \text{ or } \tau = 0 \text{ or } L(\widehat{X},X,U) = \emptyset\right\} && \text{(system operation)} \\ \mathcal{D} &:= \left\{q : V(x,S) \geq \sigma a_1 V(\hat{x},S) \text{ and } \tau = 1 \text{ and } L(\widehat{X},X,U) \neq \emptyset\right\} && \text{(episode)} \end{aligned}$$

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